Subcritical growth of long cracks in heterogeneous ceramics

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This paper models the subcritical growth of long cracks in ceramic structures containing heterogeneities. In such cases, the microstructure is believed to promote deflection of the crack. Due to this, the following effects are observed: (i) there is scatter in the K_I–V data; (ii) N, the subcritical crack growth susceptibility coefficient, as obtained from specimens with long cracks, erroneously shows large values as compared to specimens with small flaws; and (iii) the lifetimes predicted from long-crack experiments (especially in the double torsion load relaxation mode) are more than the predictions from short cracks. All of the effects mentioned above are explained by the present model.

1. Introduction

The subcritical crack growth phenomenon in brittle ceramics is a handicap for their use as a structural material. Because of the subcritical crack growth, the material may fail months after the first application of the load. Usually three kinds of test are carried out to understand this slow crack growth behaviour [1]. The first kind of crack growth test uses fracture mechanics samples containing large cracks. In such tests the crack growth rate V is measured as a function of the applied stress intensity factor K_I, thereby generating the so-called K_I–V curve. It is generally assumed that the crack growth velocity V and the applied stress intensity factor K_I, have a power-law relationship as shown in the equation

$$V = AK_I^N \tag{1}$$

where A and N are constants. N is known as the subcritical crack growth susceptibility coefficient. The extent of subcritical crack growth is large in materials having small N values and vice versa.

In the second kind of test, samples (either containing indentation cracks or machining flaws) are fractured using various constant stressing rates. It has been shown in the literature [1] that an estimate of N can be obtained from the above data. Information obtained from both of these tests can lead to predictions of the lifetime of the structure in question. In the third kind of test, specimens are stressed under a dead-weight load and the time to fracture is noted. Although this test directly produces the lifetime data, carrying it out is difficult because of the long times taken in some cases. Therefore, the first two types of test are often employed in calculating lifetimes. These tests are performed assuming that crack growth parameters are independent of the specimen geometry and microstructure. But it has been shown by Pletka and Wiederhorn [2, 3] that the above assumptions do not hold good for heterogeneous structures. N values determined from specimens having large cracks are significantly higher than those evaluated from specimens with short cracks. It is believed that this effect is a manifestation of the microstructure, so microstructure and specimen geometry are actually interrelated. Long cracks which are longer than the scale of the microstructure obviously have a different growth behaviour as compared to the short cracks, which are of the same size as the scale of the microstructure. Attention must therefore be paid to the crack paths in the experimental samples during crack growth tests.

This question was re-examined by Cook and co-workers [4–6] by performing very elegant indentation tests. They varied the indentation load to produce crack sizes which are comparable to the scale of the microstructure and greater than that. Thus, variation of indentation load produced cracks in both short and long crack-length regimes. The data showed that as the indentation load is increased, the toughness increases and finally reaches a constant value. This behaviour was rationalized in terms of a grain-localized apparent R-curve function [4, 6]. Further, stressing-rate tests (2nd kind) were carried out to check what effect the R-curve had on the subcritical crack growth behaviour. The results showed that in stressing-rate experiments, long and short cracks give rise to similar N values.

Okada and Sines [7] examined the path of crack growth in the case of a delayed failure test (3rd kind). By careful usage of a dye penetrant, they established that failure takes place due to the growth and coalescence of very small flaws. Based on these experimental observations, they proposed a model of coalescence of cracks to predict the delayed failure of the material. The model described [7] can also lead to an explanation of the scatter seen in the K_I–V data observed in polycrystalline materials such as glass–ceramics and PZT [8, 9].

So far, not much has been reported in the literature about the crack paths during the subcritical growth of long cracks in ceramic samples. Pletka and Wiederhorn

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TABLE I Comparison of salient features of the present paper with previous references

Sample & testing condition	Present work	Cook *et al.* [4, 5]	Okada and Sines [7]
Type of experiment	Long cracks with load relaxation technique (1st kind)	Stressing rate (2nd kind)	Dead-weight loading (3rd kind)
Crack size	Long cracks only	Range of cracks both long and short	Short cracks
Crack path geometry	Considered	Not considered	Considered
Difference in N value from long to short crack growth regime	Explains the over-estimation of N value compared to short-crack data	N value remains constant for a wide size range of cracks	Does not consider N value

[3] presented micrographs of crack paths in different ceramic systems. It is evident from their micrographs that the crack path is not straight, but zigzag in nature. The intent of the present paper is to present a phenomenological model which takes into account this zigzag behaviour of the crack path and to explain the scatter in the K_I-V data for various ceramic systems. We have chosen several systems where it is believed that such a type of crack growth can occur in specimens with large cracks. Three different materials that are under consideration here have a wide size-range of heterogeneities.

In order to point out important differences between the present work and that of others [4, 5, 7], we have compared them in Table I. The present paper considers crack growth behaviour in specimens with long cracks [1st kind of test]. From this viewpoint and others listed in Table I, it seems that the present model is complementary to the other models. In order to understand the subcritical crack growth phenomenon in ceramics, all of these models have to be compared.

2. The model

Since the model is based on the observations made by Pletka and Wiederhorn [3], we should bring out the salient features of their findings. This will enable us to set the guidelines for the model. First, Pletka and Wiederhorn's data were collected using the double torsion (DT) load relaxation technique. It is generally agreed that DT is a Mode I type of specimen. Even though Mode I loading is applied to the specimen, the microstructure promotes mixed-mode crack growth at the tip. This is evident from the micrographs of alumina having different grain sizes. Secondly, it is believed that because of the zigzag path there is a wide scatter in the data. In fact, in alumina with an average grain size of $16\,\mu$m it was very difficult to obtain reliable data because of the erratic crack growth [3]. Again the micrograph of the crack path will support this conjecture.

In the literature, such zigzag crack growth phenomena are given the general name "crack deflection". Faber and Evans [10] presented the first-ever model to predict the increase in toughness by deflection. Suresh [11] applied the idea of crack deflection to examine fatigue crack growth in metals. We will follow his approach to treat slow crack growth in ceramics.

As discussed by Suresh [11], there could be several configurations of the deflected crack paths. The crack could be forked, kinked, twisted etc. However, the main configurations are (i) tilted crack (Fig. 1a) and (ii) twisted crack (Fig. 1b). Suresh considered crack growth by the tilting process, which is easy to model. On the other hand, Faber and Evans [10] proposed a generalized mechanism of crack deflection by tilting and twisting. To simplify the model, we will use only tilting.

Following Lawn and Wilshaw [12], we will now obtain the stress intensity factors for a tilted crack (Fig. 1a). The crack is shown to be tilted from the OX axis at an angle θ at O′, the tip of the main crack. Mode I loading is applied at O. The transformed stress intensity factors can be simply obtained by coordinate transformation from Cartesian to polar. The normal

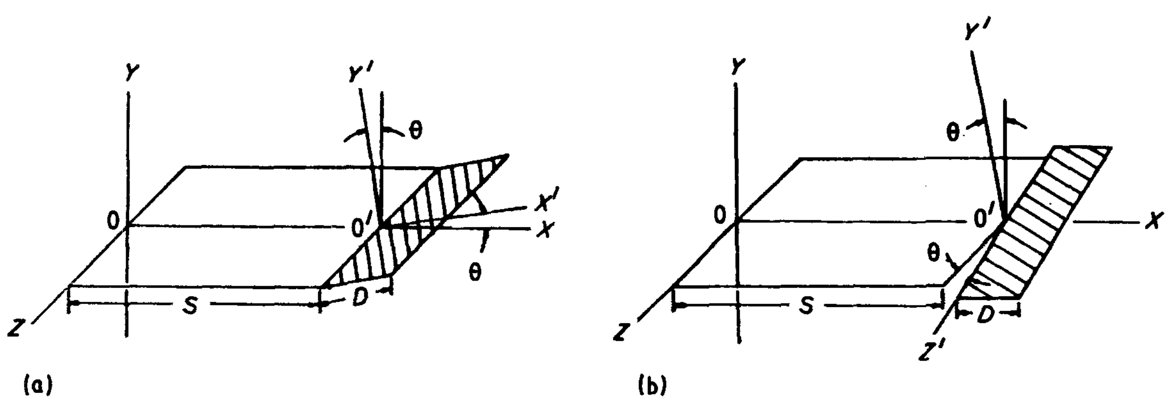


Figure 1 The two basic crack deflection mechanisms: (a) tilt mechanism, (b) twist mechanism.

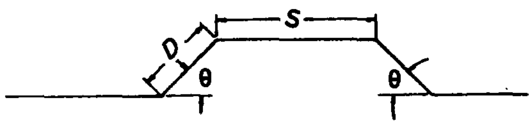


Figure 2 The geometry of a deflected crack path.

and shear components are given as

$$\begin{aligned}\sigma_{Y'Y'} &= \sigma_{\theta\theta} = \frac{K_I}{(2\pi r)^{1/2}}\cos^3\frac{\theta}{2}\\ \sigma_{XY} &= \sigma_{r\theta} = \frac{K_I}{(2\pi r)^{1/2}}\sin\frac{\theta}{2}\cos^2\frac{\theta}{2} \qquad (2)\\ \sigma_{XZ} &= 0\end{aligned}$$

The transformed stress intensity factors can be written as

$$\begin{aligned}K_I'(\theta) &= K_I\cos^3\frac{\theta}{2}\\ K_{II}'(\theta) &= K_I\sin\frac{\theta}{2}\cos^2\frac{\theta}{2}\end{aligned} \qquad (3)$$

The idealized crack path is shown in Fig. 2, where S is the length of the straight crack and D is that of the deflected crack; K_I applies to crack growth in S and an effective K_I applies to crack growth in D. Assuming a simple co-planar strain energy release rate, this effective K_I can be written as $(K_I'^2 + K_{II}'^2)^{1/2}$. Hence the average stress intensity factor during crack growth is

$$\bar{K}_I = \frac{(K_I'^2 + K_{II}'^2)^{1/2}D + K_I S}{D + S} \qquad (4)$$

From Equations 2 and 3

$$\bar{K}_I = \frac{K_I[D\cos^2(\theta/2) + S]}{D + S} \qquad (5)$$

The average velocity is given by

$$\bar{V} = \left(\frac{D\cos\theta + S}{D + S}\right)V \qquad (6)$$

These parameters can then be fed into the typical slow crack growth relationship

$$\bar{V} = A(\bar{K}_I)^N \qquad (7)$$

where A is a constant and N is the slow crack growth exponent. From Equations 5, 6 and 7

$$V = A\left(\frac{D\cos\theta + S}{D + S}\right)^{-1}\left(\frac{D\cos^2(\theta/2) + S}{D + S}\right)^N K_I^N \qquad (8)$$

Note that in Equation 6, A and N are material parameters. Whereas Equation 7 is the averaged-out $K_I V$ relationship, Equation 8 is valid for a particular deflection condition. The averaged data should consist of many such deflections during the growth of the crack. These deflection conditions are defined by the ratios $D/(D + S)$ and the θ values. In this paper we judiciously choose these conditions and plot K_I–V data for a particular condition. Finally, we compare these theoretical curves with the experimentally collected data. This sort of comparison shows which deflection conditions are probable in a given system.

3. Results

The results of this paper are based on Equation 8 and are applied to various materials. We have grouped them according to the scale of their microstructure. We have considered cordierite-based glass–ceramics with a wide range of microstructures (typical crystallites between 1 and 5 μm), an intermediate-grained alumina with an average grain size of 9 μm, and an alumina-based refractory with a grog size of about 10^{-3} m. The experimentally obtained K_I–V data were fitted to straight lines to generate A and N as mentioned in Equation 8. Based on the microstructure, some values of D, S and θ are chosen.

3.1. *Cordierite-based glass–ceramics*

This material has been extensively studied [13]. Further, Baskaran *et al.* [14] describe an interesting study of slow crack growth data and fracture toughness data related to the crystallite sizes produced by different heat treatments. The heat treatments typically include a nucleation heat treatment at 820° C for 2 h, followed by growth treatments at 1260° C for different periods of time. Table II lists these conditions along with the fitted values for A and N. Although several SEM micrographs and fractographs are shown by Baskaran *et al.* [14], no crack path is shown. In the absence of an actual crack path we have to choose a proper $D/(D + S)$ ratio. Initially, $D/(D + S)$ is chosen as 1/3 and θ is varied between 0 and 45° in the steps of 15°. Angles more than 45° seem to be improbable because it leads to an unusually high scatter in the data. Fig. 3 shows the modified K_I–V curves for heat treatments at 1260° C for 15 min, 1 h, 8 h, 24 h and 72 h.

3.2. Intermediate grain-sized alumina

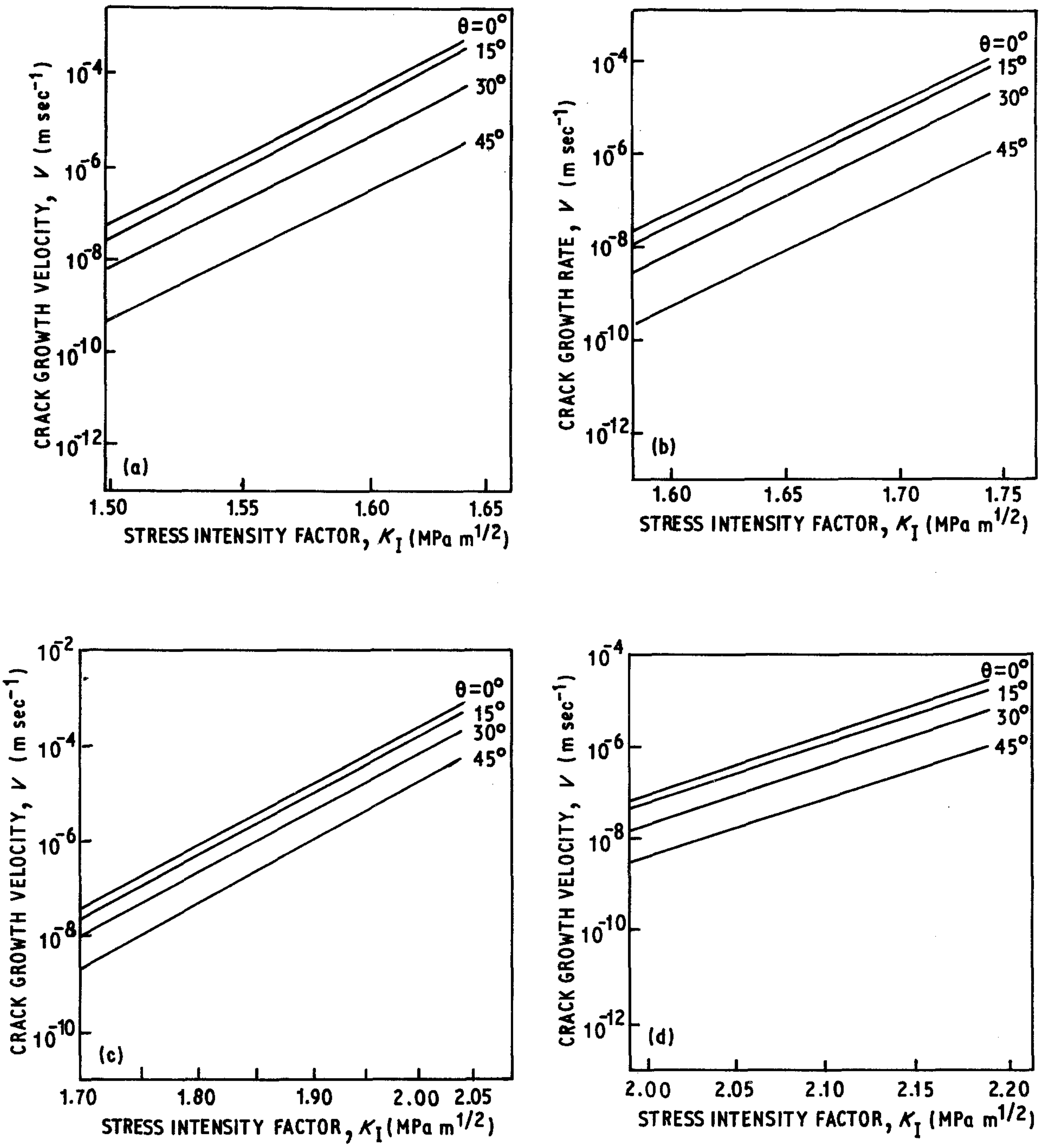
The data for this material are taken from Pletka and Wiederhorn [3]. In terms of the microstructure, this material has uniform grains as opposed to the previous material which, in fact, is a glass–crystal composite. The intermediate grain-sized alumina has a somewhat coarser grain size (average grain size $\simeq 9\,\mu$m) as compared to the previous material. So this material presents a microstructure of intermediate size. Fig. 4 shows the K_I–V curve for $D/(D + S) = 1/3$ and $\theta = 0$, 15, 30 and 45°. These graphs were plotted using $N = 99.1$ and $A = 1.48 \times 10^{-62}$.

3.3. Alumina refractories

This material represents a microstructure with coarse heterogeneities with grog sizes $\sim$1 mm [15]. Fig. 5 shows the K_I–V data for $D/(D + S) = 1/3$ and

TABLE II Subcritical crack growth parameters for cordierite glass–ceramics, heat-treated at 1260° C for various times

Heat treatment duration	Fracture toughness (MPa m$^{1/2}$)	A	N
15 min	2.11	2.72×10^{-25}	98.90
1 h	2.20	1.53×10^{-27}	95.80
8 h	2.25	1.53×10^{-27}	55.80
24 h	2.52	2.02×10^{-27}	65.03
72 h	2.16	3.87×10^{-31}	112.00



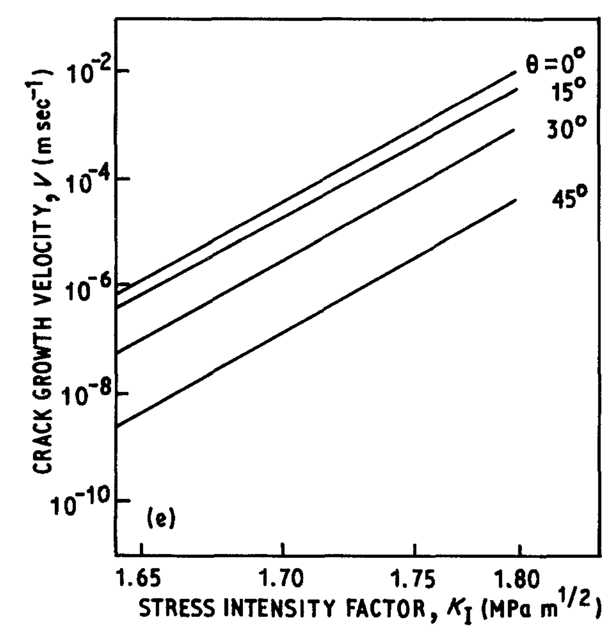


Figure 3 K_I–V data for cordierite-based glasses heat-treated at 1260° C for (a) 15 min, (b) 1 h, (c) 8 h, (d) 24 h and (e) 72 h. $D/(D + S) = 1/3$.

$\theta = 0$, 15, 30 and 45°. N is assumed to be 69.4 and A is equal to 2.38×10^{-9}.

4. Discussion

4.1. Crack growth in heterogeneous as compared to homogeneous materials

The model considers the growth of large cracks in heterogeneous materials. Heterogeneities are supposed to promote the so-called crack growth by deflection. Compared to heterogeneous materials, the crack growth occurs in a straight-line fashion in a homogeneous material. Precisely for the same reason, there will not be any difference in crack growth behaviour in large and small cracks. This leads to similar values of N, as determined from specimens with large cracks

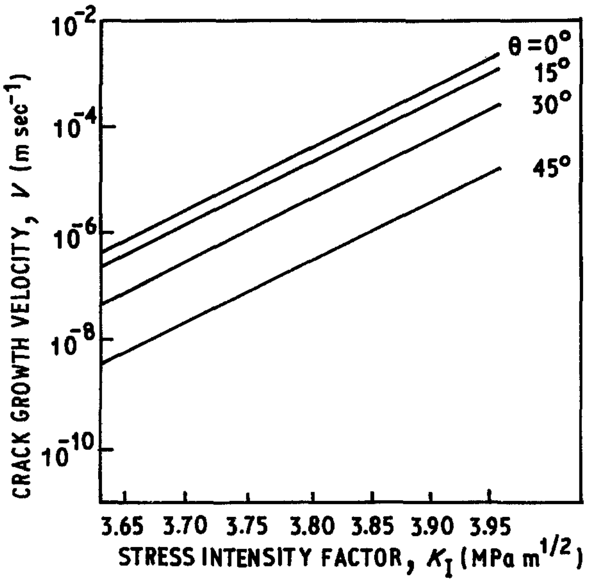


Figure 4 K_I–V data for intermediate grain-sized alumina (9 μm). $D/(D + S) = 1/3$.

and the ones with short cracks. This conjecture is proved to be true in the case of ultralow-expansion glass [3].

4.2. Effect of varying $D/(D + S)$ ratio and θ

So far, we have only presented results with a constant $D/(D + S)$ ratio and varying angles of deflection. Fig. 6 shows a typical plot for intermediate grain-sized microstructure with $\theta = 30°$ and $D/(D + S) = 0$, 1/2, 1/3, 1/4 and 1/5; A and N are chosen to be the same as those that were used for Fig. 4. It is observed from the plots that with constant $D/(D + S)$ and increasing angles of deflection the K_I–V curves shift to lower velocity values for the same K_I. The effect is the same when the extent of deflection $D/(D + S)$ is increased, keeping θ as constant. Both the effects can be rationalized by using Equation 8. Physically speaking, the former case occurs when the heterogeneities are of uniform size and spaced close by each other.

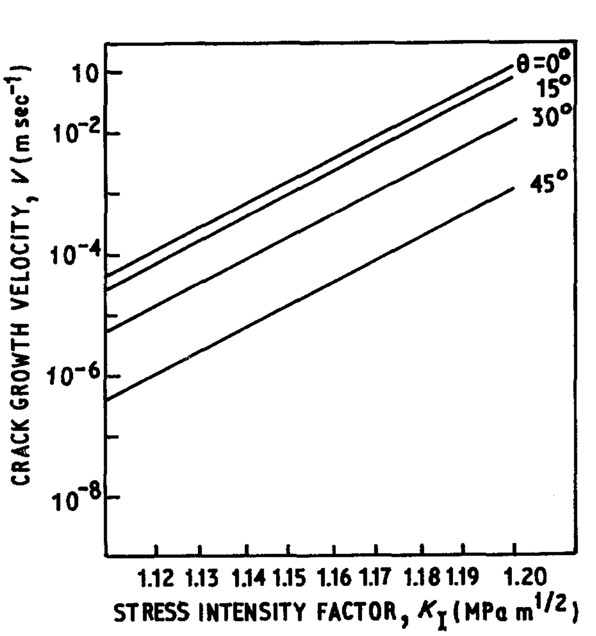


Figure 5 K_I–V data for the high alumina refractory. $D/(D + S) = 1/3$.

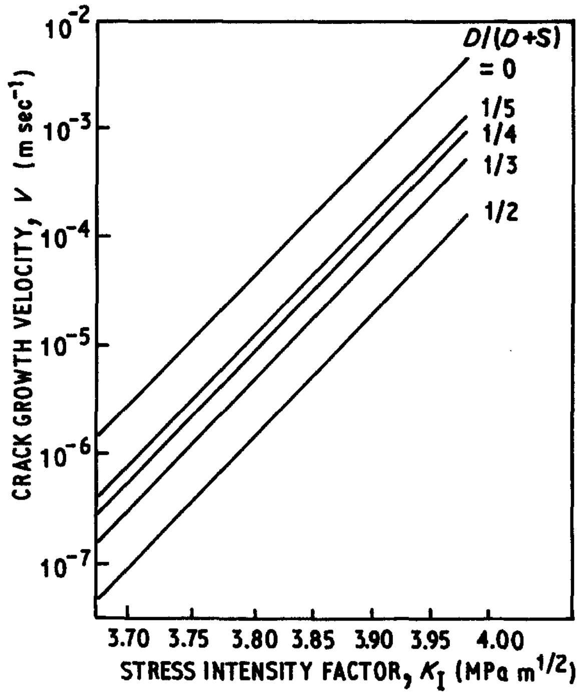


Figure 6 Effect of varying $D/(D + S)$ ratio on K_I–V data for intermediate grain-sized alumina (9 μm) at $\theta = 30°$.

This is so in the case of glass–ceramics as well as intermediate grain-sized alumina and the alumina refractories. On the other hand, deflection at constant angle with varying $D/(D + S)$ may not occur in a physical system. However, even in a physically occurring system the crack growth may not be as simple as we have modelled, but rather a mixture of both of the processes. So the parameters $D/(D + S)$ and θ will change continuously as the crack is deflected by the microstructure.

We suggest that henceforth one should be careful in obtaining crack growth data for heterogeneous materials. A lot of attention must be paid to the actual crack path. From the crack paths the average values of $D/(D + S)$ and θ values are to be calculated, taking into consideration all the deflections that occurred during the crack growth. This kind of averaging procedure has been carried out by Faber and Evans [10].

4.3. Test technique and crack deflection

This topic can be subdivided between specimens with long cracks and those with short cracks. As noted in Section 1, long cracks can be defined as those that geometrically extend through several heterogeneities. Short cracks, on the other hand, have dimensions the same as that of the heterogeneities. One reason why the N values from the two types of specimen are different is argued to be the deflection of longer cracks around microstructure. However, Freiman *et al.* [16] have shown that in constant-moment double cantilever beam specimens the N values obtained are similar to those obtained from the stressing-rate experiments. The reason for this is that in constant-moment specimens the crack is under a constant driving force, as opposed to the relaxation specimens where the driving force decreases as the crack grows. A constant driving force lets the crack propagate through the heterogeneities, though with a reduced speed. In relaxation experiments the crack does not have that much driving force, especially towards the end of the relaxation. Thus, even though the crack is straight at the beginning of relaxation, it may deflect substantially at the end of the relaxation. Therefore, the crack deflection model can be properly applied to DT relaxation data where the propensity for crack deflection is greater. In the present paper, data were selected involving only DT load relaxation tests. Hence there is justification for applying the model to these cases.

In samples with short cracks, the interaction of the heterogeneities with the crack is very much less. This is due to the nature of growth of the short crack. Since the short cracks have less energy, they usually stop when they encounter a tough heterogeneity. Instead some other crack starts growing.

4.4. Relationship between N and microstructure

In this section we discuss how the deflection crack growth can lead to an increase in N value in specimens having long cracks. Fig. 7 shows the K_I-V curves of glass–ceramics heat-treated at 1260° C for 8 h. The reason for choosing this material is because this is the commercial heat treatment. The conditions used here are $D/(D + S) = 1/3$ and $\theta = 0, 6, 12, 18, 24$ and 30°. Because of crack deflection, the actual K_I-V curve will be somewhere in between. In a typical DT load relaxation test the crack has a higher driving force at the beginning, so the probability of undergoing deflection is less and the crack will not deflect initially. On the other hand, deflection should be greater towards the end of the experiment (i.e. lower crack velocity and K_I). The crack path microstructures presented by Pletka and Wiederhorn [3] do not reveal much regarding this postulate. However, the SEM fractographs of glass–ceramics presented by Baskarau *et al.* [14] clearly show that at lower K_I and V values the crack has undergone deflection. We now assume that the crack follows a certain K_I-V curve, and then due to deflection there is a sudden transition to a lower curve. The tendency will increase as we go down the K_I-V curve. The solid curve labelled "average" in Fig. 7 depicts this hypothetical curve and N is calculated to be 67.8.

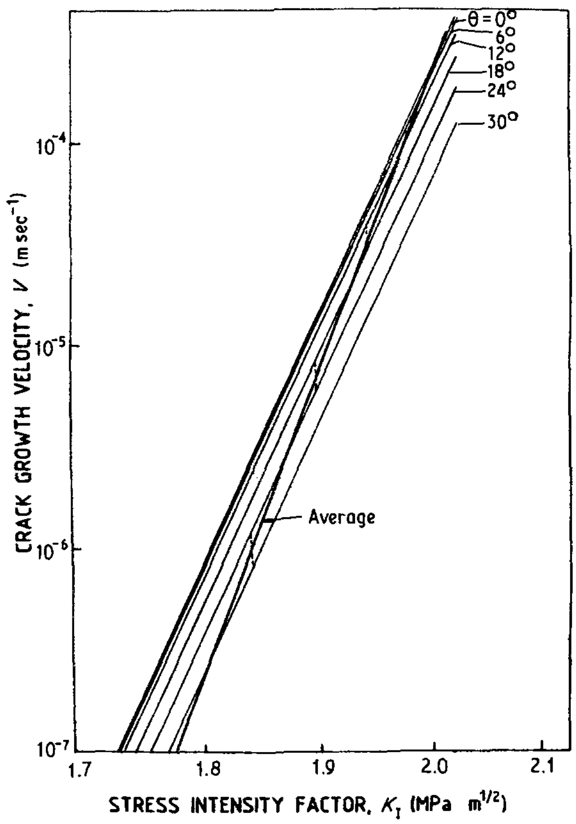


Figure 7 Simulation of actual crack deflection growth to show increase in N. Intermediate grain-sized alumina (9 μm). $D/(D + S) = 1/3$.

4.5. Lifetime prediction

The consequence of crack deflection is a change in the lifetime of the structure. To determine the lifetime we consider the equation

$$K_I = \sigma_a Y a^{1/2} \tag{9}$$

where K_I is the applied stress intensity factor, σ_a is the applied stress, Y is a geometrical constant and a is the flaw size. By differentiating Equation 8 with respect to time,

$$\frac{dK_I}{dt} = \left(\frac{(\sigma_a Y)^2}{2K_I}\right) V \tag{10}$$

and substituting the crack growth relationship of Equation 8 and integrating, we have for the lifetime t_f the expression

$$t_f = 2K_I^{2-N} - K_{IC}^{2-N} \Bigg/ \left[(\sigma_a Y)^2 \left(\frac{D\cos\theta + S}{D + S}\right)^{-1} \times \left(\frac{D\cos^2(\theta/2) + S}{D + S}\right)^{N} A(N-2) \right] \tag{11}$$

In Equation 11 the crack deflection effect is introduced by the factor

$$\left(\frac{D\cos\theta + S}{D + S}\right)\left(\frac{D\cos^2(\theta/2) + S}{D + S}\right)^{-N}.$$

Assuming $2D = S$ and the N value of commercially heat-treated material, the above factor is calculated to be 1.37, 3.49 and 15.973 for deflection angles $\theta = 15$, 30 and 45°, respectively, thus reflecting the increase in lifetime due to crack deflection.

Perhaps it is not out of the way to point out that short cracks behave differently. As discussed by Okada and Sines [7], the short cracks propagate catastrophically. The lifetime consists of the time of growth of short cracks according to a microscopic law followed by the time of growth of a large coalesced crack according to a macroscopic law [7]. A particular situation will control which of the aforesaid processes is the deciding factor. It is shown in the example given by Okada and Sines [7] that both of the processes are important. Thus, in the growth of large cracks deflection is important whereas in case of small flaws, growth and coalescence are important.

In view of the above discussion, it is noted that lifetime predictions using DT relaxation experiments may lead to higher predictions, because the mode of crack growth will be different from what happens in a real situation.

4.6. Crack deflection compared with other mechanisms

Throughout this paper we have emphasized the fact that long cracks grow by deflection, but it is worth noting that in some ceramic materials the resistance to crack growth increases with growth of the cracks [17–19]. This is known as *R*-curve behaviour. This behaviour is seen during the growth of a short crack to a long crack [4–6] and also during the growth of cracks in intermediate grain-sized alumina [20–22]. The second case is pertinent to the present discussion. Two mechanisms have been proposed for this behaviour: (i) a microcrack zone ahead of the crack tip [23], and (ii) the bridging of unbroken ligaments behind the crack tip [22, 24].

We note that during a subcritical crack growth test the extent of crack growth is 100% of the starting crack. This extent of crack growth also takes place during R-curve determination tests. Ligament bridging can explain R-curve behaviour. Therefore, it may be likely that ligament bridging is taking place in intermediate grain-sized alumina during slow crack growth. One would expect K_I-V data with higher slopes together with high N values. However, more experiments and modelling are required to quantify this statement. Becher and Ferber [25] argued that subcritical crack growth is affected by the residual grain boundary stresses which are dependent on the grain size. However, if the grain size is greater than the critical grain size for microcracking, shielding of crack tip stresses occurs leading to an almost constant N value. Thus microcracking does not explain the observed results *per se*. For glass–ceramics there are no reported data on R-curve behaviour, so the present data can be explained only in terms of crack deflection.

Finally, R-curve behaviour is an effect of non-linear behaviour at the crack tip [23]. During the initial period of crack growth, there is a small zone at the crack tip. Since in the present case the K_I values are lesser than K_{IC}, it can be assumed that the zone around the crack tip is small in size. R-curve behaviour is therefore expected to be small. Hence, as of now, the present model of crack deflection can suitably explain the experimental data.

5. Conclusions

This paper focuses on the behaviour of long cracks in heterogeneous ceramics. From the microstructures of crack paths presented in the literature, it was concluded that the crack undergoes deflection during subcritical crack growth. Two most important crack deflection configurations are the tilt and twist modes. In the present model a simplifying assumption was made by considering only the tilt mode. Calculations show that deflection causes substantial scatter in the K_I-V data, and predict a systematic shift of N value towards higher values. The above conclusion is true for heterogeneities of different sizes (10^{-6} to 10^{-3} m). It is shown that data taken from DT relaxation experiments will lead to higher predicted lifetimes.

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